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The linear stability in systems with intensive mass transfer—III. Liquid–liquid

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Abstract—A linear analysis of the stability of the flow in a laminar boundary layer under conditions of intensive interphase mass transfer between two liquids, when high mass fluxes through the phase boundary induce secondary flows, is suggested. The first liquid is in motion over the second one (in rest). Hydrodynamic stability in the two phases is considered. The critical Reynolds numbers in the first place are obtained at different intensities of non-linear mass transfer in the laminar boundary layer. The influence of the direction of the intensive interphase mass transfer on the hydrodynamic stability is analogous to the cases where phase boundary is motionless, but depends on the distribution of the diffusive resistance in the two phases. The motion of the interface is considerably more intensive than the one in the gas–liquid system, which leads to an increase in the stability of the flow to a large degree. The flow is stable at a large Reynolds number in the second phase. Copyright © 1996 Elsevier Science Ltd.

1. INTRODUCTION

The first two reports [1, 2] show that the motion of the interface [2] influences sufficiently the hydrodynamic stability of a flow in the gas boundary layer on the boundary with the flat liquid surface. In addition, this motion and the effect of the intensive interphase mass transfer [1] are superposed. This effect must be amplified considerably under conditions of intensive interphase mass transfer between two liquids, where the hydrodynamic interaction between them is stronger and surface velocity is higher.

Non-linear effects in the case of an intensive interphase mass transfer between two liquids can manifest themselves with the same intensity in both phases. In a number of extraction processes, where the motion of one of the phases (dispersion environment) induces motion in another (dispersion phase), these effects are of great interest. Further, we can consider the hydrodynamic stability under conditions of an intensive interphase mass transfer between two liquid phases, where the velocity in the volume of one of them is zero (Fig. 1).

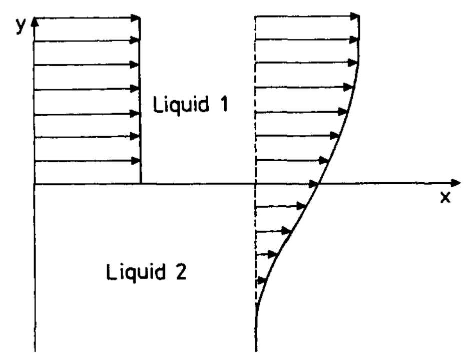

Fig. 1. Velocity profiles in liquid flows in the boundary layer (liquid–liquid system).

2. VELOCITY PROFILES IN THE BOUNDARY LAYER

The mathematical model of non-linear mass transfer in liquid–liquid systems, where the first liquid is in motion over the second one (which is in rest) can be obtained from the model 'liquid–gas' [2]. It is necessary to have the diffusion equation for the second phase, and the following must be added into the boundary conditions:

$$x = 0, \quad (y \to -\infty), \quad u_2 = 0; \quad y = 0, \quad c_1 = mc_2;$$

$$y = 0, \quad v_j = -\frac{MD_j}{\rho_{0j}^*}\frac{\partial c_j}{\partial y}, \quad j = 1, 2;$$

$$y = 0, \quad \frac{\rho_1^* D_1}{\rho_{10}^*}\frac{\partial c_1}{\partial y} = \frac{\rho_2^* D_2}{\rho_{20}^*}\frac{\partial c_2}{\partial y}, \qquad (1)$$

where m is the distribution coefficient and the indexes 1, 2 denote liquid 1 and liquid 2, respectively.

The problem formulated above was solved numerically [3, 4] and the boundary values for the velocity and its derivatives are obtained. This gives us an opportunity in the analysis presented here to generate the velocity profiles by the following hydrodynamic problem:

NOMENCLATURE

- A dimensionless wave number, initial value of the Blasius function
- B initial value of first derivative of the Blasius function
- c concentration
- C dimensionless phase velocity, initial value of second derivative of the Blasius function
- D diffusion coefficient
- f Blasius function
- k parameter
- m diffusion coefficient
- M molecular mass
- Re Reynolds number
- u velocity of basic stationary flow in x direction
- v velocity of basic stationary flow in y direction
- x coordinate
- y coordinate.

Greek symbols

- ε parameter
- θ parameter
- ν kinematic viscosity
- ξ variable
- ρ density.

Subscripts and superscripts

- * conditions on interface
- 0 conditions in volume
- 1 liquid 1 phase
- 2 liquid 2 phase
- cr critical number
- i imaginary part of complex number
- max maximum
- min minimum
- r real part of complex number.

$$u_j\frac{\partial u_j}{\partial x}+v_j\frac{\partial u_j}{\partial y}=\nu_j\frac{\partial^2 u_j}{\partial y^2},\quad \frac{\partial u_j}{\partial x}+\frac{\partial v_j}{\partial y}=0;$$

$$x=0,\quad u_1=u_0,\quad u_2=0;$$

$$y=0,\quad u_j=u_{j0},\quad v_j=v_{j0},$$

$$\frac{\partial u_j}{\partial y}=R_j,\quad j=1,2, \qquad (2)$$

where u_{j0}, v_{j0} and R_j $(j=1,2)$ are determined in ref. [4].

Introducing similarity variables

$$u_j=u_0 f_j'(\xi_j),\quad v_j=\left(\frac{u_0\nu_j}{4x}\right)^{0.5}(\xi_j f_j'-f_j),$$

$$\xi_j=(-1)^{j-1}y\left(\frac{u_0}{\nu_j x}\right)^{0.5},\quad j=1,2, \qquad (3)$$

leads to problem which allows us to determine the velocity profiles

$$2f_j'''+f_jf_j''=0,$$

$$f_j(0)=A_j,\quad f_j'(0)=B_j,\quad f_j''(0)=C_j,$$

$$j=1,2\quad (f_1'(\infty)=1,\quad f_2'(\infty)=0) \qquad (4)$$

where A_j, B_j and C_j are results of the numerical solution [4], and they are displayed in Table 1.

The velocity profiles $f_j'(\xi_j)$ $(j=1,2)$ depend substantially on the effect of the non-linear mass transfer $(A_j, j=1,2)$, which is characterized by the parameters θ_j $(j=1,2)$ [3, 4]

Table 1. The computed values of A_j, B_j, C_j and k in cases where the resistance to diffusion is limited by the mass transfer in the continuous phase $(m/b=0, \theta_1=\theta, \theta_2=0)$. The second part of table—these values in cases of commensurate diffusional resistances $(b/m=1, \theta_1=\theta_2=\theta)$

	θ	A_j	B_j	C_j	k
$\varepsilon=10$	−0.5	0.66525	0.439	0.26565	0.673
$m/b=0$	−0.3	0.032988	0.420	0.26565	0.747
	−0.1	0.0094	0.405	0.26565	0.805
	0	0	0.4	0.26565	0.823
	0.1	−0.008261	0.394	0.26565	0.846
	0.3	−0.022194	0.384	0.26565	0.883
	0.5	−0.033445	0.3755	0.26565	0.915
$b/m=1$	−0.5	0.02117	0.4132	0.26565	0.773
	−0.3	0.012867	0.4075	0.26565	0.8
	−0.1	0.004316	0.402	0.26565	0.82
	0	0	0.4	0.26565	0.823
	0.1	−0.004316	0.3967	0.26565	0.836
	0.3	−0.012867	0.39	0.26565	0.862
	0.5	−0.02117	0.385	0.26565	0.88

$$\theta_j=\frac{M(mc_{20}-c_{10})}{\rho_{j0}^{*}m^{j-1}},\quad j=1,2. \qquad (5)$$

This effect is superposed with the effect of the hydrodynamic interaction between the phases $(C_j, j=1,2)$. Hence, the interface velocity $(b_j, j=1,2)$ takes into account both of the above mentioned effects.

3. RESULTS AND DISCUSSION

The linear analysis of the hydrodynamic stability in the liquid–liquid systems is made analogously to the

Table 2. Values of the critical Reynolds numbers Re_{cr} wave velocities C_r, wave numbers A and $C_{r\,min}$, A_{min} obtained (in cases $m/b = 0, \theta_1 = \theta, \theta_2 = 0$ and $b/m = 1, \theta_1 = \theta_2 = \theta$)

ε	θ	Re_{cr}	A	C_r	A_{min}	$C_{r\,min}$
$\varepsilon = 10$	−0.5	3145	0.315	0.6235	0.358	0.6246
$m/b = 0$	−0.3	2663	0.320	0.6155	0.364	0.6163
	−0.1	2343	0.325	0.6092	0.372	0.6101
	0	2243	0.330	0.6081	0.372	0.6085
	0.1	2145	0.320	0.6042	0.374	0.6053
	0.3	1983	0.320	0.5997	0.375	0.6009
	0.5	1859	0.330	0.5969	0.377	0.5974
$b/m = 1$	−0.5	2503	0.325	0.6130	0.367	0.6135
	−0.3	2398	0.325	0.6099	0.370	0.6111
	−0.1	2288	0.325	0.6079	0.371	0.6086
	0	2243	0.330	0.6081	0.372	0.6085
	0.1	2170	0.330	0.6064	0.374	0.6066
	0.3	2079	0.320	0.6020	0.375	0.6036
	0.5	1999	0.325	0.6008	0.375	0.6015

one in the case of gas–liquid systems [2]. The velocity profiles (4) are introduced into the Orr–Sommerfeld equation. The results obtained show that the stability of the profiles depends considerably on the non-linear effects of the mass transfer θ_j ($j = 1,2$), as well as on the interface velocity B_j ($j = 1,2$).

In the cases, where the non-linear effects are not presented ($\theta = \theta_1 = \theta_2 = 0$) the increase of the interface velocity (B_j ($j = 1,2$)) leads to significant stabilizing of the flow (Figs. 2, 4).

The effect of the non-linear mass transfer in the liquid 1 (Table 2, $m/b = 0$) and the effects of the increase of interface velocity are superposed and their total influence on the stability of the flow in phase 1 is shown in Figs. 2–4.

Under the conditions of commensurable diffusive resistances in the two liquids (Table 2, $m/b = 1$) the non-linear effects decrease (Figs. 5, 6). The influence of the non-linear effects (θ) on the stability of the flow decreases.

Linear analysis of the hydrodynamic stability of the phase 2 shows analogous results to those in ref. [2]. The flow is stable with large Reynolds numbers ($Re \approx 25\,000$), which can be explained with the profile shape (approximately the same as the Couette one, Fig. 7).

The investigations of the hydrodynamic stability in the systems with intensive interphase mass transfer show that the stability increases with the rise of the interface velocity and the rise of concentration gradients in the cases of interphase mass transfer directed from the volume to the phase boundary. The decrease of the interface velocity and the change of the direction of interphase mass transfer destabilize the flow in the boundary layer.

The experimental researches [5–7] of the mass transfer in systems with intensive interphase mass transfer between two liquids show in a number of cases a higher mass transfer rate, compared with the cases which are predicted by linear theory of mass transfer. So far it was explained by the Marangoni effect, i.e. the creation of interfacial tension gradients as a result of temperature or concentration heterogeneity on the phase boundary. The interfacial tension gradient induces secondary flows directed tangentially to the phase boundary. They change the velocity profiles in the boundary layer. Thus, the mass transfer rate is directly affected. In the case of hydrodynamic instability of the new profiles the flow spontaneously evolves from laminar into turbulent and the mass transfer rate drastically increases.

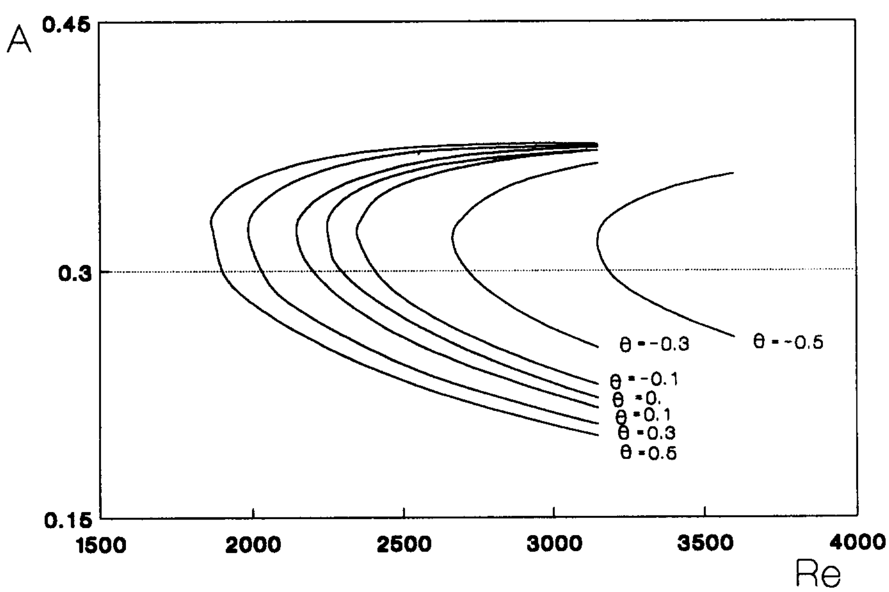

Fig. 2. The neutral curves for the wave number A as a function of the Reynolds number Re in liquid 1 ($m/b = 0, \theta_1 = \theta, \theta_2 = 0$).

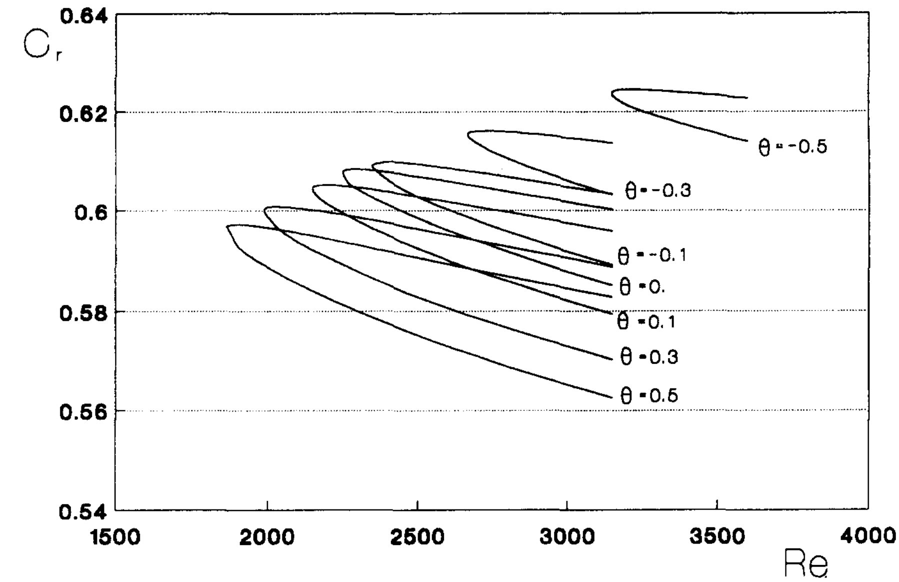

Fig. 3. The neutral curves for the phase velocity C_r as a function of the Reynolds number *Re* in liquid 1 ($m/b = 0$, $\theta_1 = \theta$, $\theta_2 = 0$).

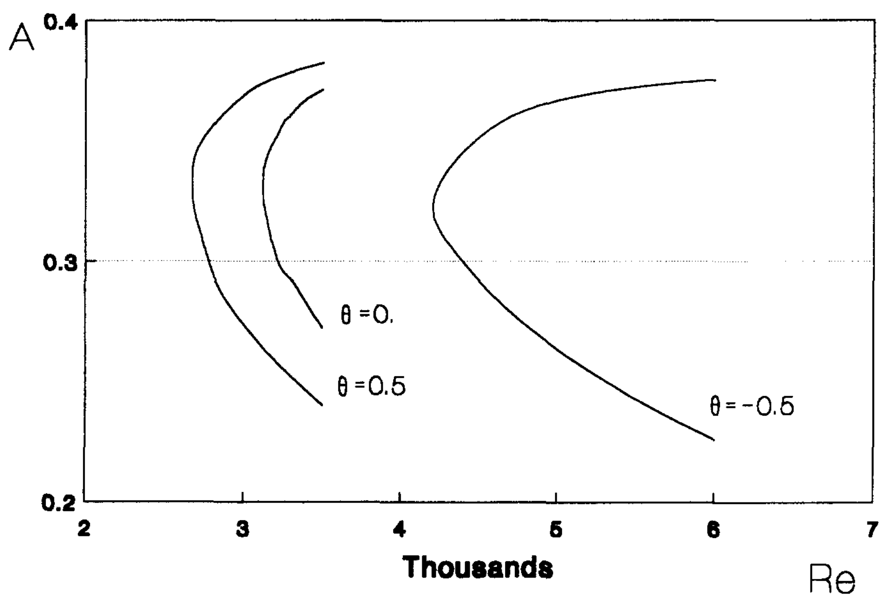

Fig. 4. The neutral curves for the wave number *A* as a function of the Reynolds number *Re* in liquid 1, where interface velocity increases ($m/b = 0, \theta_1 = \theta$, $\theta_2 = 0$).

The results obtained in this work show that under the conditions of intensive interphase mass transfer, high mass fluxes induce secondary flows directed normally to the phase boundary. These secondary flows change the velocity profiles, consequentially they change the kinetics of the mass transfer (non-linear

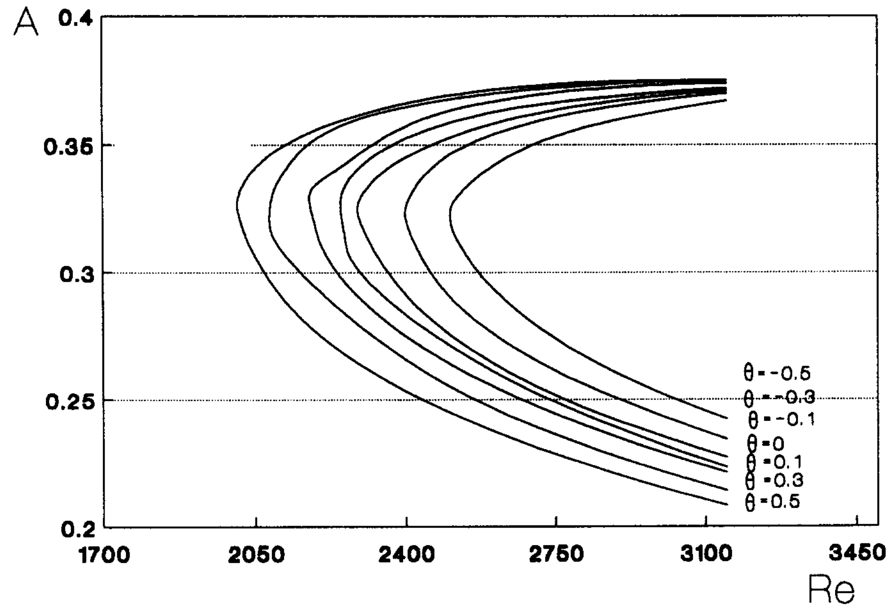

Fig. 5. The neutral curves for the wave number A as a function of the Reynolds number Re in liquid 1, ($b/m = 1$, $\theta_1 = \theta_2 = \theta$).

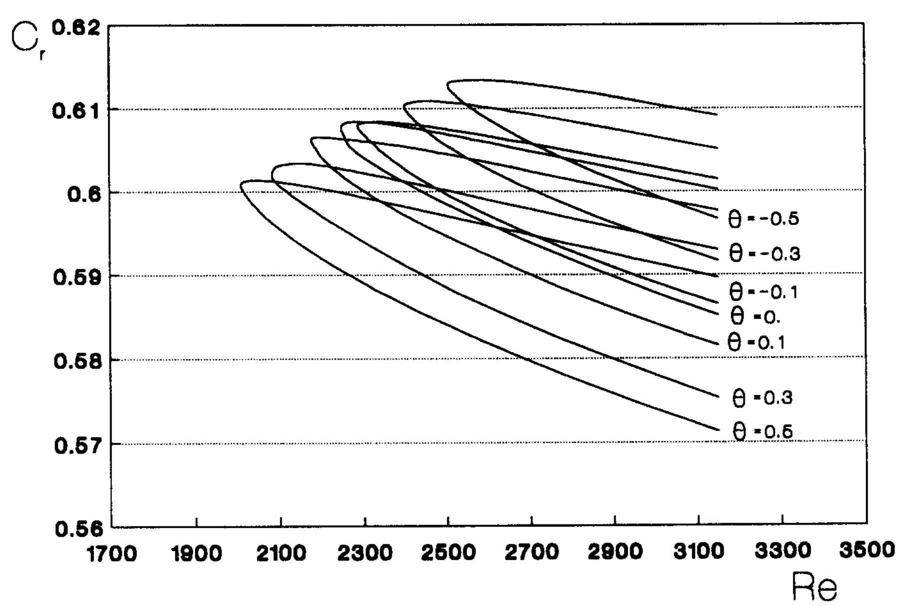

Fig. 6. The neutral curves for the phase velocity C_r as a function of the Reynolds number Re in liquid 1 ($b/m = 1$, $\theta_1 = \theta_2 = \theta$).

mass transfer) and the hydrodynamic stability of the flow. This is a radically different mechanism for the influence of intensive interphase mass transfer on the kinetics of mass transfer and the hydrodynamic stability in liquid–liquid systems.

The theoretical results obtained allow comparative

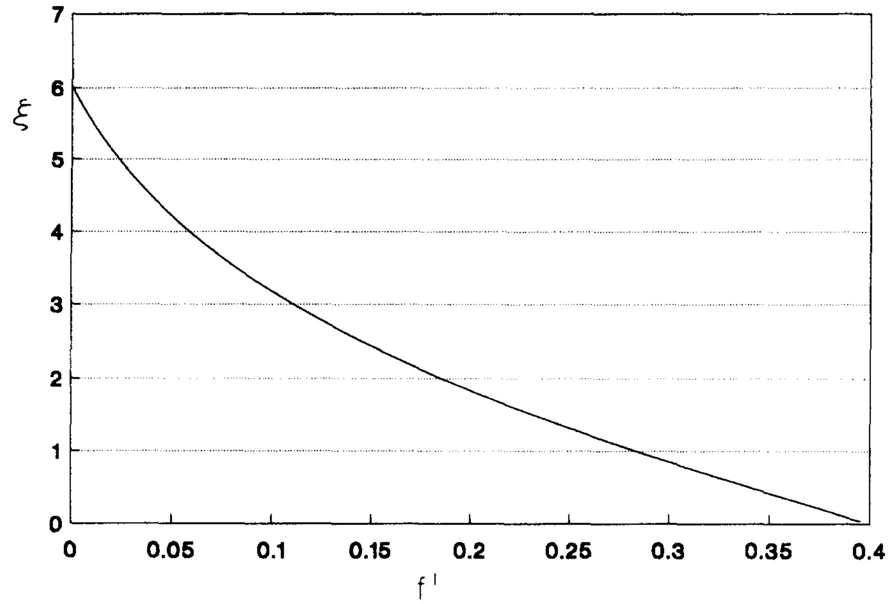

Fig. 7. Velocity profile of the flow of liquid 2 ($\theta_2 = 0, \varepsilon_2 = 20$).

analysis of the influence of the Marangoni effect and the effect of non-linear mass transfer on the mass transfer rate, and the hydrodynamic stability of systems with intensive interphase mass transfer.

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